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AIAA 82-4292

Oblique Impingement of a Round Jet on a Plane Surface

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Introduction

THE oblique impingement of a round jet on a plane surface establishes a three-dimensional flowfield with properties of relevance to the analysis of VTOL aircraft in-ground effect. The azimuthal distribution of momentum flux resulting from such an impact cannot be found solely on the basis of global momentum theorems (e.g., Taylor[1]). Truncated Fourier series models,[2] as well as detailed numerical analyses,[3] have been used to represent this distribution for VTOL applications. The former technique provides useful estimates, while the latter technique, though correct in principle, suffers from computational resolution problems and the inherent inability to provide a simple, easily used formula.

Direct momentum efflux measurements of a three-dimensional submerged, jet impingement flowfield are made difficult to achieve by the effects of entrainment.[4] Taylor[5] used colliding water jets to measure directly the efflux distribution for jet inclinations of $\theta=\pi/6$, $\pi/4$, and $\pi/3$ and uncovered several global features of the flow. He noted that, within the thin collision sheet, the flow was very nearly radial from some point within the impact region, except perhaps at the most shallow of inclinations (i.e., $\theta=\pi/6$). Many years earlier, Schach[6] measured the upstream and downstream efflux resulting from round water jets impinging upon a plane wall. Over the entire range of incidence angle (viz., $\theta=\pi/6$, $\pi/4$, $\pi/3$, $5\pi/12$, and $\pi/2$), he found that his data was in excellent agreement with a rather simple model that he had devised.

The purpose of this Note is to present a simple expression for the azimuthal redistribution of impingement momentum flux. This expression in deduced from the Schach model and accurately predicts the observations made by Taylor.

Schach's Model

Schach considered a uniform, inviscid and incompressible jet of unit radius impinging with angle θ upon a plane surface (Fig. 1). The stagnation streamline resides in the symmetry plane and, away from the stagnation point, it is parallel to the jet axis separated by the distance b. The intersection of the stagnation line with a plane normal to the jet axis (viz., the jet plane) forms the origin for angle β, where β is measured from the symmetry plane. Flow through the region $-\pi/2<\beta<\pi/2$ is deflected downstream and flow through the region $\pi/2<\beta<3\pi/2$ is deflected upstream. If the distance from the origin to the jet edge is denoted by $r(\beta,b)$, then

$$r(\beta,b)=b\cos\beta+\sqrt{1-b^2\sin^2\beta} \tag{1}$$

and the momentum flux through the elemental area included by $d\beta$ is

$$dJ=\tfrac{1}{2}\rho u^2 r^2(\beta,b)\,d\beta$$

where ρ and u are the jet density and velocity, respectively, and the jet momentum flux J is

$$J=\rho u^2\int_0^\pi r^2(\beta,b)\,d\beta=\pi\rho u^2$$

The efflux far from the stagnation point recovers to the jet velocity and is assumed to spread along surface rays defined by the azimuthal angle ϕ. Here, ϕ is measured from the symmetry plane with origin at the stagnation point (Fig. 1). The ray $\phi=\pm\pi/2$ divides the efflux into its upstream ($x<0$) and downstream components. Schach reasoned that, since the jet plane forms an angle $(\pi/2-\theta)$ with the surface plane (Fig. 2), the angle β undergoes a change to angle $\phi(\beta)$ given by

$$\tan\phi=\sin\theta\tan\beta \tag{2}$$

Conservation of x momentum demands that

$$J\cos\theta=2\int_0^\pi\cos\phi\,dJ=\frac{J}{\pi}\int_0^\pi r^2(\beta,b)\cos\{\phi(\beta)\}\,d\beta \tag{3}$$

Received Nov. 9, 1981; revision received April 28, 1982.

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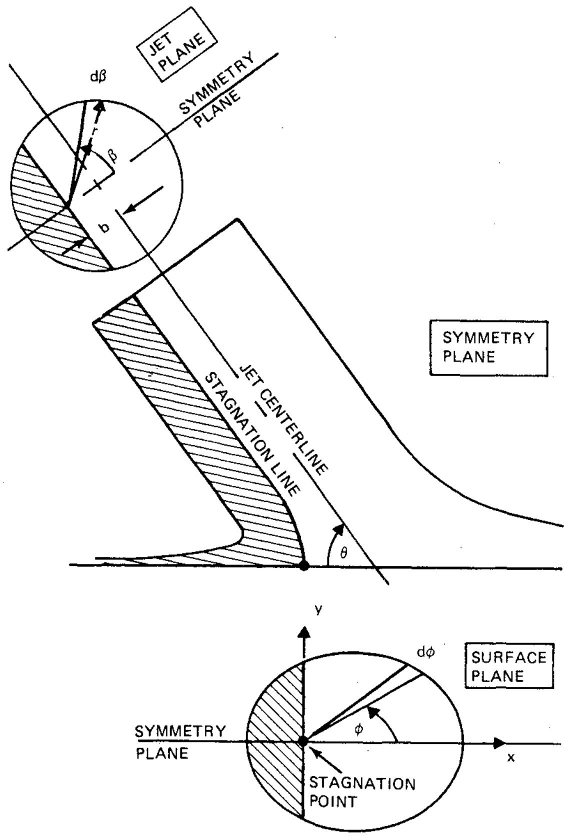

Fig. 1 Schematic for impingement model (shaded zone is deflected upstream).

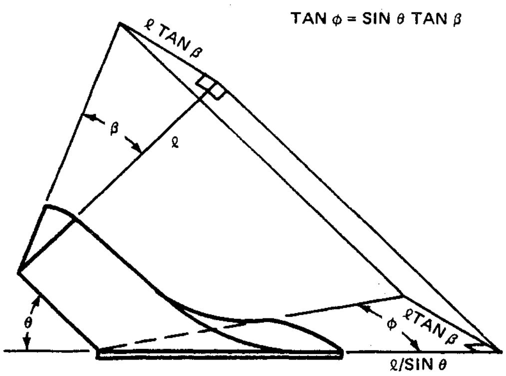

Fig. 2 Relationship between jet plane and ground plane angles.

and Schach showed that Eqs. (1-3) yield

$$\int_0^\pi \frac{\cos^2\beta\sqrt{1-b^2\sin\beta}}{\sqrt{1-\cos^2\theta\sin^2\beta}}\,\mathrm{d}\beta = \frac{\pi}{2b}\cos\theta$$

which is satisfied by

$$b=\cos\theta \tag{4}$$

Hence, the fraction of the flow carried downstream is

$$\frac{1}{\pi}\int_0^{\pi/2} r^2(\beta,\cos\theta)\,\mathrm{d}\beta = 1-\frac{\theta}{\pi}+\frac{1}{2\pi}\sin 2\theta \tag{5}$$

Table 1 shows that when this quantity is extracted from Taylor's data[5] it is in agreement with Schach's work.

Table 1 Downstream fraction of total flow

Jet angle θ	Schach[6] Eq. (5)	Schach[6] Experiment	Taylor[5] experiment
$\pi/3$	0.805	0.819	0.801
$\pi/4$	0.909	0.912	0.906
$\pi/6$	0.971	0.974	0.961

Table 2 Experimental peak—internal force distribution[5]

θ	$(2\pi/J)I(\pi/2,\theta)$
$\pi/3$	1.053
$\pi/4$	1.028
$\pi/6$	1.198

Results

Define $E(\phi,\theta)$ as the distribution function for radial momentum efflux with respect to surface angle ϕ when the jet is inclined at angle θ with respect to the surface and the jet velocity has been recovered. If the distribution is normalized by $E(\phi,\pi/2)=J/2\pi$, then

$$\frac{E(\phi,\theta)}{E\left(\phi,\frac{\pi}{2}\right)} = r^2[\beta(\phi),\cos\theta]\frac{\mathrm{d}\beta}{\mathrm{d}\phi}(\phi)$$

and using Eqs. (1) and (2), it can be shown that

$$\frac{2\pi}{J}E(\phi,\theta)=\frac{\sin^3\theta}{(1-\cos\theta\cos\phi)^2} \tag{6}$$

This relatively simple expression is nothing more than the normalized distribution of area for the ellipse projected by the jet cross section onto the surface plane (Fig. 1).

A comparison of Eq. (6) with the azimuthal distributions measured by Taylor[5] shows good agreement over the entire range of data (Figs. 3a-c). The greatest disagreement occurs when $\theta=\pi/6$ and Taylor suggests that the flow may no longer be radial at this shallow incidence.

With the distribution function in hand, it is a simple matter to calculate the ratio of lateral to normal reaction forces L/V, for the symmetric half-jet. Here,

$$\frac{L}{V}=\frac{\int_0^\pi E(\phi,\theta)\sin\phi\,\mathrm{d}\phi}{\frac{1}{2}J\sin\theta}=\frac{\sin^2\theta}{\pi}\int_0^\pi\frac{\sin\phi}{(1-\cos\theta\cos\phi)^2}\,\mathrm{d}\phi=\frac{2}{\pi} \tag{7}$$

so that Schach's model predicts that L/V is the same invariant with respect to impingement angle as observed by Taylor.[1,5]

Finally, consider that the internal force parallel to the surface required to orient each elemental segment of efflux is[5]

$$E(\phi,\theta)\{(\cos\phi-\cos\theta)^2+\sin^2\phi\}^{1/2}\,\mathrm{d}\phi$$

and acts in the direction χ, where

$$\tan\chi=\sin\phi/(\cos\phi-\cos\theta) \tag{8}$$

Define the distribution of these internal forces with respect to their direction of action as $I(\chi,\theta)$ so that

$$I(\chi,\theta)=E[\phi(\chi),\theta]\{[\cos\phi(\chi)-\cos\theta]^2+\sin^2\phi(\chi)\}^{1/2}\frac{\mathrm{d}\phi}{\mathrm{d}\chi}(\chi) \tag{9}$$

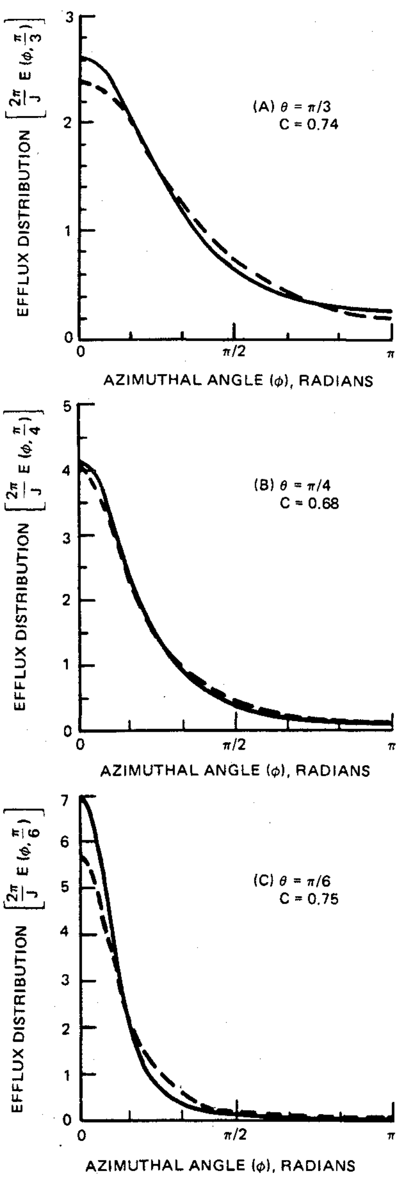

Fig. 3 Azimuthal distribution of efflux momentum;--- Taylor[5] experiment (C is inferred discharge coefficient), ——Schach[6] model [Eq. (6)].

Using Eqs. (6) and (8) in conjunction with Eq. (9) yields

$$\frac{2\pi}{J} I(\chi,\theta) = \frac{\sin^3\theta}{(1-\cos^2\theta\sin^2\chi)^{3/2}} \tag{10}$$

which shows that the normalized distribution is symmetric about $\chi = \pi/2$ for all values of θ. This, too, is in agreement with Taylor's observations.[1,5] In addition, it is apparent that the peak magnitude of the normalized internal force distribution is given by $(2\pi/J)I(\pi/2,\theta) = 1$ and is independent of incidence angle. Table 2 demonstrates that Taylor's[5] data are again consistent with the Schach model except, perhaps, at $\theta = \pi/6$.

Conclusions

The model of Schach for round jet impingement yields a useful expression [Eq. (6)] for the azimuthal distribution of momentum efflux that compares well with data[5,6] over the range $\pi/6 \leq \theta \leq \pi/2$. Moreover, the model satisfies subtle characteristics of the flowfield uncovered by Taylor. In particular, the lateral to normal reaction forces on the symmetric half-jet are in the ratio of $2/\pi$, independent of jet incidence angle. Also, the distribution of internal forces parallel to the surface are in accord with his observations.

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AIAA 81-1871R

Turbulent Boundary-Layer Flow over Re-entry Bodies Including Roughness Effects

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Nomenclature

C_f = skin-friction coefficient
k = roughness height
Pr = Prandtl number
R = radius
Re = Reynolds number
S = surface distance
St = Stanton number
T = temperature
ρ = density
θ = momentum thickness
τ = shear stress
ν = kinematic viscosity

Subscripts

aw = adiabatic wall
e = edge condition
i = incompressible
N = nose
w = wall condition
0 = smooth wall

Introduction

THE computation of turbulent boundary-layer flow over re-entry bodies is important in the assessment of their performance. Surface roughness greatly influences the

Presented as Paper 81-1871 at the AIAA Atmospheric Flight Mechanics Conference, Albuquerque, N. Mex., Aug. 19-21, 1981; submitted Sept. 30, 1981; revision received April 5, 1982.

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